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RAYLEIGH MOTIONS IN AN ELASTIC HALF-SPACE WITH A CONSTRAINED BOUNDARY†

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STATIONARY surface waves in elastic half-space with boundary conditions corresponding to a combination of the Winkler model and an inertial layer at the boundary are studied. It is found that the velocity of propagation of a harmonic wave depends on the frequency, and the presence of constraints in a direction normal to the boundary results in stopping of the low frequencies, when the effect of elastic rigidity and inertia of the boundary are taken into account at the same time, and when the inertia of the support has no effect. The frequencies are not stopped when the displacements along the boundary are restricted and when the influence of elastic rigidity on the normal displacements of the boundary is neglected.

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1. NORMAL CONSTRAINTS

We consider, in an elastic half-space $y \geqslant 0$ with velocities of propagation of the longitudinal and transverse waves a and b, the Rayleigh motion defined by the potentials

$$\varphi(x, y, t) = A \sin \omega\xi e^{-\omega\alpha y}, \quad \psi(x, y, t) = B \cos \omega\xi e^{-\omega\beta y} \tag{1.1}$$

$$\xi = \frac{x}{p} - t, \quad \alpha = \sqrt{\frac{1}{p^2} - \frac{1}{a^2}}, \quad \beta = \sqrt{\frac{1}{p^2} - \frac{1}{b^2}}, \quad A, B = \text{const}$$

Here $p \leqslant b$ is the velocity of wave propagation and ω is the oscillation frequency. Here the coordinates of the displacement vector and the stress tensor components are

$$u = \frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y} = (p^{-1}Ae^{-\omega\alpha y} - \beta Be^{-\omega\beta y})\,\omega \cos \omega\xi$$

$$v = \frac{\partial\varphi}{\partial y} - \frac{\partial\psi}{\partial x} = (-\alpha Ae^{-\omega\alpha y} + p^{-1}Be^{-\omega\beta y})\,\omega \sin \omega\xi$$

$$\sigma_{xy}/(\mu\omega^2) = (-2\alpha p^{-1}Ae^{-\omega\alpha y} + (\beta^2 + p^{-2})Be^{-\omega\beta y})\cos \omega\xi \tag{1.2}$$

$$\sigma_{yy}/(\mu\omega^2) = ((\beta^2 + p^{-2})Ae^{-\omega\alpha y} - 2\beta p^{-1}Be^{-\omega\beta y}) \sin \omega\xi$$

where μ is the shear modulus of the half-space.

The boundary conditions have the form (k_1 is the rigidity of the foundation and m_1 is inertial resistance)

$$\sigma_{xy} = 0, \quad \sigma_{yy} = k_1 v - m_1 \partial^2 v/\partial t^2 \quad \text{when} \quad y = 0 \tag{1.3}$$

Substituting the values of the stresses into Eqs (1.3), we arrive at a system of two homogeneous linear equations for determining A and B. The condition for the existence of a non-trivial solution leads to the following equation in $q \equiv p/b \leqslant 1$:

$$Q_1(q) = (k_1/\omega - m_1\omega)b/\mu \tag{1.4}$$

Here

$$Q_1(q) = R(q)/W_1(q), \quad R(q) = (2 - q^2)^2 - 4\sqrt{1 - r^2q^2}\sqrt{1 - q^2}$$

$$W_1(q) = q^3\sqrt{1 - r^2q^2}, \quad r = b/a$$

and R is the left-hand side of the Rayleigh equation for a half-space with a free boundary.

The function $Q_1(q)$ increases monotonically from zero when $q_R = (c_R/b) < 1$ (c_R is the velocity of the Rayleigh wave in a free half-space), to the value $Q_1(1) > 0$ bounded for real values of $r \in [0, 1]$. The solid lines in Fig. 1 represent this function, the lines 1–4 corresponding to $r = 0.1$; 0.5; 0.7; 0.9. Also, $q \to q_R (p \to c_R)$ as $\omega \to \infty$ and Rayleigh-type motion will not exist when $[(k_1/\omega) - m_1\omega]b/\mu \leqslant Q_1(1)$. The latter quadratic inequality has two roots for every fixed value of r, and one of these roots is $\omega_k > 0$. For the frequencies $\omega \leqslant \omega_k$ the Rayleigh solution is not realized (low frequencies are stopped). Stopping of low frequencies ($\omega \leqslant k_1 b/(\mu Q_1(1))$) also occurs when $m_1 = 0$. If $k_1 = 0$, then there is no stopping of the frequencies and the wave velocities are lower than the Rayleigh velocities for all ω.

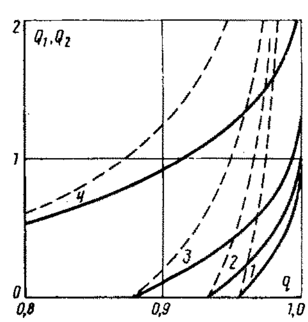

Fig. 1.

For a fixed value of the ratio wave velocities r the possible velocities of propagation of the Rayleigh motion over the range from $q = q_R$ to $q = 1$. Below we give the values of q_R and $Q_1(1)$ for several values of r for materials of solids (the velocity q was assumed to be equal to the Rayleigh velocity q_R when the value of R did not exceed 10^{-5})

r	0.1	0.3	0.5	0.7	0.9
q_R	0.955	0.949	0.932	0.878	0.608
$Q_1(1)$	1.005	1.048	1.155	1.400	2.294

2. TRANSVERSE CONSTRAINTS

The boundary conditions

$$\sigma_{xy} = k_2 u - m_2 \partial^2 u / \partial t^2, \quad \sigma_{yy} = 0 \text{ when } y = 0 \tag{2.1}$$

$$Q_2(q) = (k_2/\omega - m_2\omega), \quad Q_2 = R/W_2, \quad W_2 = q^3\sqrt{1-q^2} \tag{2.2}$$

The function $Q_2(q)$ increases monotonically from $-\infty$ when $q = 0$, to $+\infty$ when $q = 1$ (the dashed lines 1–4 in Fig. 1 for $r = 0.1$; 0.5; 0.7; 0.9), so that (2.2) has a real root for any k_2 and m_2 and we have no frequency stopping. Also, $q = q_R$ at $\omega = \omega_0 \equiv \sqrt{(k_2/m_2)}$ and $q < q_R$ at $\omega > \omega_0$. The velocity of propagation is lower than the Rayleigh velocity for all ω, provided that $k_2 = 0$.

3. MIXED WINKLER FOUNDATION

The boundary conditions

$$\sigma_{yy}(x, 0, t) = k_1 v(x, 0, t), \quad \sigma_{xy} = k_2 u(x, 0, t)$$

after substituting expressions (1.2) into them and equating to zero the determinant of the system of linear homogeneous equations for the constants A and B, lead to the following equation for determining the relative velocity q:

$$R(q) - \frac{bq^3}{\mu\omega}(k_1\sqrt{1-r^2q^2} + k_2\sqrt{1-q^2}) - \frac{k_1k_2}{\mu^2\omega^2}(1 - \sqrt{1-r^2q^2}\sqrt{1-q^2}) \tag{3.1}$$

When $q_k < q < 1$, the discriminant of Eq. (3.1) is positive, and one of the roots $\omega > 0$. An analogous argument holds when the elastic stiffness and inertia of the support on the boundary are both taken into account.

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